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Analysis of OH-6A Helicopter Flight Test Data Using Lissajous Figures

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This paper demonstrates that a new application of Lissajous figures to flight test data gives different capabilities to identify both the commensurateness and the relative phase between measured quantities. These are not available from either conventional time or frequency domain flight test data analysis.

Introduction

A NEW application of Lissajous figures is presented in this paper. Flight test data from a helicopter that had higher harmonic control, an active vibration suppression system, were used in the analysis.

Lissajous Figures for Flight Test Data Analysis

In Lissajous figures, when a point executes two motions simultaneously at right angles to one another in a plane, it traces out a two-dimensional trajectory that lies within a rectangle. If there is no relation between the two motions, the point does not return to its original position, and its trajectory fills up the rectangle by its repeated passages. However, recognizable stationary patterns emerge whenever the two motions are either of the same frequency or are commensurate. Two motions are commensurate with each other if the ratio of their frequencies is a rational number and the initial phases are a simple fraction of 2π. Commensurateness does not require that either motion be absolutely periodic (constant frequency). These stationary patterns are Lissajous figures named after J. A. Lissajous, who made an extensive study of such motions.[1]

Figure 1 shows commensurate motions at two different combining frequencies and five different initial phases. The motions combined at right angles to each other are both simple harmonics, and their frequencies ratio is a rational number. Different initial phase differences are readily apparent by inspection. Further, the trajectory is tangential to the boundary lines of the rectangle enclosing the Lissajous figure. Inspection of the ratio of the number of tangencies made by the trajectories with the vertical and horizontal adjacent sides of the rectangle gives the ratio of the combining frequencies. For example, at $\omega_1/\omega_2 = 4$ and $\phi = 90$, inspection reveals four tangencies on either vertical side and one tangency on either horizontal side of the rectangle, which means the frequencies are at a ratio of 4. The only exception to this rule is when the initial phase is either 0 or 180 deg.

Historically, Lissajous figures have been used extensively in electrical and mechanical engineering. A signal generator generates one signal of known frequency, and an oscilloscope compares the known signal with an unknown by combining them at right angles to each other. If there is a small difference between the frequencies, one of the patterns in Fig. 1 is maintained for a few oscillations. It will gradually change to another pattern as the phase difference between the orthogonal vibrations changes. If the frequencies are ω_1 and ω_2, then one motion gains $\omega_1 - \omega_2$ periods/s on the other. Hence, the cycle of patterns repeats after $1/(\omega_1 - \omega_2)$ s. Timing the repetition cycle of the patterns accurately gives the difference in the two frequencies. Lissajous figures used this way are very valuable in comparing frequencies, calibrating frequency sources, and obtaining the natural frequencies of mechanical components.

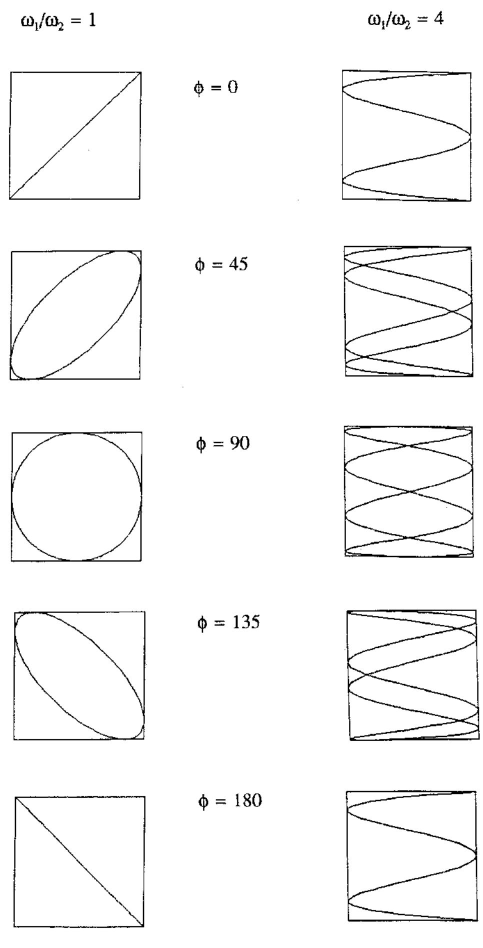

Fig. 1 Lissajous figures. Frequency ratios of 1:1 and 4:1 with initial phase differences of 0, 45, 90, 135, and 180 deg.

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Another historical application for Lissajous figures has been helicopter rotor trim. In steady flight, a helicopter must be in equilibrium with respect to the forces and moments about three orthogonal axes at its c.g. The rotor must also be in equilibrium despite the unsymmetrical airflow caused by forward flight. Until the advent of high-speed computer methods, rotor trim was solved by hand using a graphical method based on Lissajous figures.[2]

The new application of Lissajous figures presented here is based on the old helicopter rotor trim method. Instead of plotting calculated data as in the old method, actual flight test data form the Lissajous figures. These figures plot flight test data that represent the input or forcing function along the vertical axis. They plot the output or response along the horizontal axis. Recognizable stationary patterns emerge whenever the two motions are commensurate. An immediate benefit derived from the Lissajous method is the physical insight that they offer.

Applications to Higher Harmonic Control Flight Test Data

Flight test data from an OH-6A helicopter used in a higher harmonic control (HHC) flight test program demonstrate the features and significances of using Lissajous figures. HHC was first successfully demonstrated in a joint McDonnell Douglas/NASA/U.S. Army test program from 1982 to 1984.

Forward flight of a helicopter causes its rotor blades to experience a constantly changing airflow. The velocity is greater on the blades advancing in the direction of flight and less on the retreating side. The angle of attack on the retreating side must be greater than the advancing side for balanced flight. This unsymmetrical velocity pattern causes the rotor blades to vibrate periodically at the rotor rotational rate and its higher harmonics. The rotor blades vibrate the fuselage at their attachment to the rotor hub. However, the rotor acts as a filter, allowing only certain blade harmonics to pass through the rotor hub to the fuselage. One per revolution, or 1/rev, are the vibrations at the rotor rotational rate. Higher harmonics are the multiples of 1/rev. For example, 2/rev, 3/rev, 4/rev, etc., are the second, third, and fourth harmonics, respectively. The fuselage experiences only those harmonics that are equal to a multiple of the number of rotor blades. The cause of this filtering is the periodic nature of rotor blades vibrations (1/rev, 2/rev, 3/rev, etc.) and the symmetric arrangement of the rotor blades. All other harmonics cancel each other out. In the case of the four-bladed rotor OH-6A, the only excitations reaching the fuselage are the 4/rev, 8/rev, 12/rev, etc. Of these, the 4/rev is an order of magnitude larger than the rest. Thus, the major source (about 60% of the total) of vibrations in a helicopter is essentially at a single frequency.

The HHC system is an active vibration suppression system. It continuously monitors, in the case of the OH-6A under the pilot's seat, the 4/rev vibration caused by the rotor, and actively suppresses it by exciting the swashplate at 4/rev out of phase with the rotor vibrations. The swashplate, part of the existing flight control system, normally transmits pilot control inputs from the stationary or fuselage frame to the rotor blades or rotating frame. Tilting the swashplate changes the angle of attack of the rotor blades. Reference 3 contains a review of the HHC system and flight test results.

Flight Test Data Analysis Results

We applied Lissajous analysis to the HHC flight test data using a VAX3200 workstation. Figure 2 depicts Lissajous figures for right pilot seat vertical acceleration (g) and rotor mast lateral bending (in.-lb) vs rotor blade cyclic pitch (deg). The

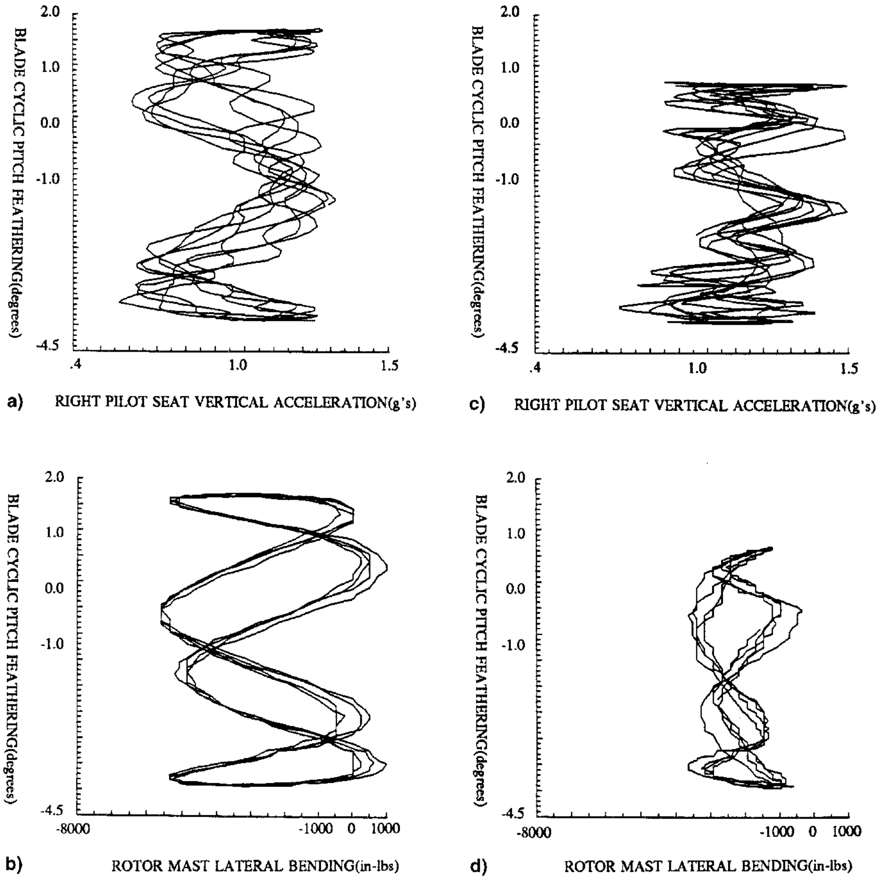

Fig. 2 Conditions are level flight at 60 kts. Lissajous of a) right pilot seat acceleration vs blade cyclic pitch feathering (HHC off); b) rotor mast lateral bending vs blade cyclic pitch feathering (HHC off); c) same as part a, except with HHC on; and d) same as part b, except with HHC on.

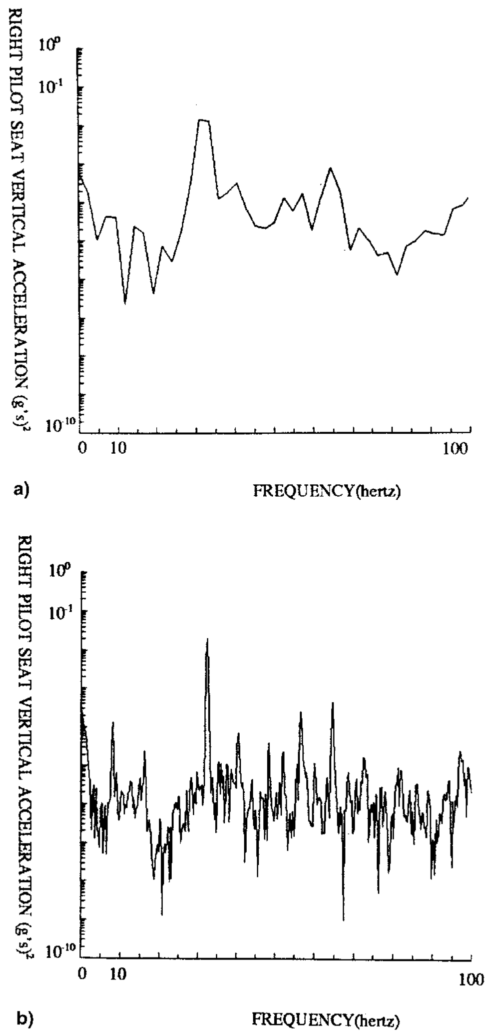

Fig. 3 Power spectral density of right pilot seat vertical acceleration: a) $\frac{1}{2}$ s of data, giving a frequency resolution of 2 Hz and b) 5 s of data giving a frequency resolution of $\frac{1}{5}$ Hz.

only condition of flight available was lateral direction HHC swashplate excitation at 60-kn level flight.

Figures 2a and 2b present data with the HHC system off. The 4/rev in the OH-6A was approximately at 32 Hz, since the main rotor rotated at about 8 Hz or 480 rpm. In Figs. 2a and 2c, the pilot seat acceleration Lissajous figure shows that the 32-Hz (4/rev) pilot seat acceleration is not commensurate with the 8-Hz (1/rev) rotor blade cyclic pitch. Figure 2a has a strong 4/rev component since it is tangent in four broad areas to the boundary lines of the rectangle that encloses it. In Figs. 2b and 2d, notice that mast bending is commensurate with blade cyclic pitch in the Lissajous figure with mast bending. Lissajous analysis can easily distinguish commensurateness of the two signals.

Figures 2c and 2d present the same data with the HHC system turned on and the HHC controller at the optimum setting. Notice that the 4/rev component in both mast lateral bending and pilot seat vertical acceleration is reduced.

Reducing the 4/rev vibrations at the rotor mast reduces the vibrations elsewhere in the fuselage. Comparing Fig. 2a to Fig. 2d shows that a substantial part of the 4/rev vibrations still exist at the rotor mast. Since rotor mast vibrations are commensurate with the blade cyclic pitch, a better location for the HHC feedback accelerometers may have been on the rotor mast. These data seem to indicate that the OH-6A location of the HHC feedback accelerometers under the pilot seat may not have been the best choice.

The HHC system attempted to reduce vibrations in the other two directions, in addition to the vertical acceleration. Although not presented here, both lateral and longitudinal pilot seat vibrations were not commensurate with the blade cyclic pitch. However, longitudinal mast bending and vertical mast excitation were commensurate with the blade cyclic pitch.

By comparing Figs. 1 and 2, the Lissajous figures clearly show relative phases between an exciting function and its response. For example, in Fig. 2b, the related phase between blade cyclic pitch and rotor mast lateral bending is approximately 120 deg.

The Lissajous figures shown used only $\frac{1}{2}$ s of flight test data. Figure 3a is a Fourier transform of only $\frac{1}{2}$ s of vertical accelerometer data, giving a frequency resolution of 2 Hz. Note the poor quality of the information in Fig. 3a compared to Fig. 3b, where 5 s of data gives a frequency resolution of $\frac{1}{5}$ Hz. Lissajous analysis works well with short data records, whereas Fourier analysis favors long data records.

Concluding Remarks

This paper presents a new application of Lissajous figures. This method graphically combines a forcing function with its response in the same figure. This article shows that by application of Lissajous figures to HHC flight test data that the following was found:

1) The commensurateness of two signals can be identified.

2) Extremely short time data records can be used.

3) Relative phase between two signals can clearly be displayed.

4) A better location for feedback sensors for HHC was indicated to be at the rotor mast.

In general, an immediate benefit derived from applications of Lissajous figures to flight test data are the physical insight that they provide for certain types of flight test data that are not available from traditional analysis methods.

Acknowledgment

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